PC Polygon Assistant

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PC Polygon Assistant is a touch-optimized Web app that displays the polygon notation for a pitch-class set. It also displays basic structural information about the set and allows the user to visually explore geometric analogies for the transposition (T) operation, inversion (I) operation, and complement (C) relation in pc space. The author created the app for his post-tonal theory students at the University of South Carolina. The app is available online at:
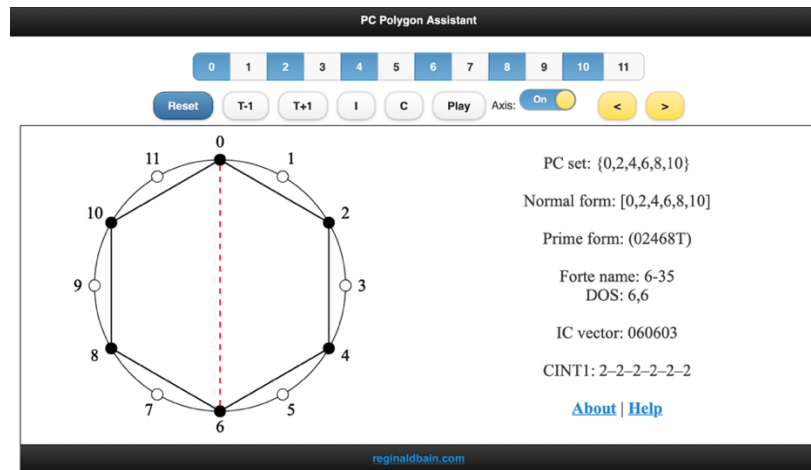
https://www.reginaldbain.com/software.html

Prerequisite
A basic knowledge of basic pitch-class set theory (e.g., Straus 1991, or Straus 2016, Ch. 1-3) is required.

QuickStart
PC Polygon Assistant's interface is shown in Figure 1.

Figure 1. PC Polygon Assistant's interface

Entering a pc set
Use the *pitch-class* (abbr. *pc*) boxes at the top of the app (labeled 0-11, where [C=0]) to enter a *pitch-class set* (abbr. *pc set*). In Figure 1, the whole-tone collection on C {0, 2, 4, 6, 8, 10} has been entered. As each new pc is added, or deleted, the app will automatically redraw the *polygon notation* (Toussaint 2013) for the current set and update its structural information. This information includes: a listing of the pc set in ascending pc order (0-11) and the set's *normal form*; the *prime form*,[1] *Forte name*, *degrees of symmetry* (abbr. DOS),[2] and *interval-class vector* (abbr. *IC vector*) for the current *set class* (abbr. *sc*). The normal form and prime form are calculated using the Rahn algorithm (Rahn 1980). Definitions of the pc set-theoretical terms above may be found in Straus 2016. CINT1 is the *cyclic interval array* of the prime form (Morris 1987), which, essentially shows the length of the sides of the polygon.

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[1] In the prime form, T=10 and E=11.
[2] DOS: x, y; where x is the degree of symmetry under T_n; and y is the degreey of symmetry under T_nI.

Transposition

The **T+1** and **T-1** buttons, called the **Transposition** buttons, will *transpose* all of the pitch classes in the current set up 1 (T_1) or down 1 (T_{11}) semitone, respectively, and then display the resultant set. For example, for the current set {0, 2, 4, 6, 8, 10}:

$$\textbf{T+1} = T_1 \{0, 2, 4, 6, 8, 10\} = \{1, 3, 5, 7, 9, 11\}$$

$$\textbf{T-1} = T_{11} \{0, 2, 4, 6, 8, 10\} = \{11, 1, 3, 5, 7, 9\}$$

Visually, transposition (T_n) is isomorphic to *rotation* of pc by 30 * n degrees in the clockwise direction.

Inversion

The **I** button, called the **Inversion** button, will *invert* all of the pitch classes in the current set about the 0-6 *axis* (or mirror) and display the resultant set.[3] For example, for the current set {0, 2, 4, 6, 8, 10}:

$$\textbf{I} = T_0 I \{0, 2, 4, 6, 8, 10\} = \{0, 10, 8, 6, 4, 2\}.$$

Visually, pc inversion (I) is isomorphic to *reflection* of a pc with respect to the 0-6 mirror.

Complement

The **C** button, or **Complement** button, calculates the current set's *literal complement* and displays the resultant set. For example, for the current set {0, 2, 4, 6, 8, 10}:

$$\textbf{C} = \text{The literal complement of } \{0, 2, 4, 6, 8, 10\} = \{1, 3, 5, 7, 9, 11\}.$$

Inversional Axis Search Tool (< >)

The Inversional Axis Search Tool is a visual aid. It allows the user to visually search a given pc set for an *axis of inversional symmetry*; i.e., a mirror about which all of the pitch classes reflect. Turn the **Axis** button on and the app will display the 0-6 axis as a red dotted line. The **Increment** (>) and **Decrement** (<) buttons allow the user to cycle through all 12 *inversional axes*.[4] Finally, when a new pc set is entered the current inversional axis is erased and the Axis feature is turned off.

Reset and Play buttons

The **Reset** button will reset will erase all current information and reset the current set to the *empty set* {}. The **Play** button will play the current pc set. Playback is based on a 4 second piano sample (A4 = 440 Hz). The app uses the following pc to 12tet pitch mappings: 0 is C4, 1 is C♯4/D♭4, 2 is D4, 3 is D♯4/E♭4, 4 is E4, 5 is F4, 6 is F♯4/G♭4, 7 is G4, 8 is G♯4/A♭4, 9 is A4, 10 is A♯4/B♭4, and 11 is B4.

Acknowledgements

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[3] Straus's I_n is equivalent to Rahn's $T_n I$ for $0 \leq n \leq 11$ (Rahn 1980).
[4] If the y component of the DOS is 0, the set it not inversionally symmetrical and no mirror exists. If y is 1, a single mirror exists. If y is greater than 1, multiple mirrors are possible.

Links
Weisstein, Eric W. "Rotation." From *MathWorld*. <https://mathworld.wolfram.com/Rotation.html>.

_____. "Reflection." From *MathWorld*. <https://mathworld.wolfram.com/Reflection.html>.

References
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Straus, Joseph N. 2016. *Introduction to Post-Tonal Theory*, 4th ed. New York: Norton.

Taylor, Benjamin, and Andrew Bernstein. 2016. "Tune.js: A Microtonal Web Audio Library." *Proceedings of the 2nd Web Audio Conference* (WAC-2016). Atlanta, GA. Available online at: <Available online at: <https://smartech.gatech.edu/handle/1853/54580>.

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