CHAPTER 3
The Frequency Domain

"[The] French scientist and mathematician Jean Baptiste Fourier (1768-1830) proved…that any *periodic waveform* can be expressed as the sum of an infinite set of sine waves. The frequencies of these sine waves must be integer multiples of some fundamental frequency."

– Burk et al., *Music and Computers*

Terms & Concepts

3.1 Frequency Domain
Domains
- Time (*a* vs. *t*)
- Frequency (*a* vs. *f*)

Envelopes
- Transients
  + Attack stage
  + Decay stage
  + Release stage
- Steady state stage
- Running average envelope
- Peak envelope

Root-mean-squared (RMS) amplitude
Running window technique
Sonogram (*f* vs. *t*)
Melograph (*pitch* vs. *t*)
Phonophotography

3.2 Phasors
Phasor representation of a sine wave
Sound analysis
Digital manipulation of sound
Sine wave model
Phasor function
Trigonometric functions
Degrees
Radians
Angular velocity of the phasor
Law of superposition (adding phasors)
Fourier's theorem
Periodic function
Fundamental frequency
Overtones, partials and harmonics
Odd-partial symmetry
Vectors
- Magnitude
- Direction

Vector addition

Sampling and Fourier Expansion
Fourier expansion
Fourier coeficients
Bins (See also § 3.4)
Waterfall 3D plot

3.3 Fourier and the Sum of Sines
Jean Baptiste Fourier (1768-1830)
Complex waveforms
Basic waveshapes
- Sine
- Sawtooth
- Square
  + Duty cycle = 0.5
  + Pulse
- Triangle

Spectrum
Infinite series
Fourier series
Fourier analysis
Fourier coefficients
- Low order
- High order

DC term
Filters (See also § 4.3)
- Low, high and band pass

Hydrophone

3.4 The DFT, FFT, and IFFT
Discrete Fourier Transform (DFT)
Fast Fourier transform (FFT)
Inverse Fast Fourier transform (IFFT)
Sample rate
Frame size (as a power of 2)
Number of bins
Bin width
Windowing
Histogram of frequencies

number of bins = Frame size / 2

bin width = f range / *# of bins*

3.5 Problems with the FFT/IFFT
Time/frequency resolution trade-off
Lobes
Time smearing

3.6 Some Alternatives to the FFT
Wavelet analysis
McAulay-Quatieri (MQ) Analysis

Software
Apple, Grapher
Desmos, GraphingCalculator
Faber Acoustic, *SignalScope*
Wolfram, Alpha

Reference

Burk, Phil, Larry Polansky, Douglas Repetto, Mary Roberts and Dan Rockmore. 2011. *Music and Computers: A Theoretical and Historical Approach*, Archival Version. Available online at: <http://music.columbia.edu/cmc/MusicAndComputers/>.