Straus Chapter 3
Some Additional Properties and Relationships

Joseph N. Straus, *Introduction to Post-Tonal Theory*, 4th ed. (New York: Norton, 2016), pp. 95-132.

TERMS & CONCEPTS

COMMON-TONE THEOREMS
Common tones under T_n (§ 3.1, p. 96)
The number of common tones under T_n may be determined from the ic vector. For example, the pc set [C, C♯, F♯, G], whose ic vector is 200022, produces the following number (#) of common tones at (@) each level of T_n:

@	T_1	T_2	T_3	T_4	T_5	T_6
#	**2**	**0**	**0**	**0**	**2**	**2** * 2
@	T_{11}	T_{10}	T_9	T_8	T_7	

Because the tritone maps onto itself under T_6, you have to multiply the ic vector's ic6 entry by 2.

Common tones under I_n (§ 3.3, p. 103)
Index matrix, or addition table
Index scoreboard
Index vector

SYMMETRY AND SET CLASS (SC)
Transpositional symmetry (§ 3.2, p. 100)
Self-mapping operation
Degree of transpositional symmetry
Transpositionally symmetrical SCs (p. 101)

Inversional symmetry (§ 3.4, p. 107)
Degree of inversional symmetry
Inversionally symmetrical SCs:
- at one level of I_n
- at more than one level of I_n (p. 111)

Intervallic palindrome, or mirror image
See also: The 12 inversional axes (p. 240)
- Mirror or *axis* name for each I_n is: $\frac{n}{2}$ to $\frac{n}{2} + 6$

Pitch space symmetry (§ 3.4.2, p. 108)
Center of symmetry and pitch centricity

Some special set classes
Augmented triad: 3-12 (048); Diminished seventh chord: 4-28 (0369); All interval tetrachords: 4-Z15 (0146) & 4-Z29 (0137); All trichord hexachord: 6-Z17 (012478); Petrushka chord: 6-30 (013679): Diatonic hexachord: 6-32 (024579), etc. See also the referential collections in Ch 5 including: 5-35, 6-20, 6-35, 7-35 & 8-28.

DOS & DF (§ 3.5, p. 112)
Degree of symmetry (DOS)
Straus (*x*, *y*) notation, where:
- x is the # of T_n self-mapping operations
- y is the # of I_n self-mapping operations

Distinct forms (DF): $DF = 24/(x + y)$

PROPERTIES & RELATIONSHIPS
Set type under:
- Cardinality (*c*)
- T_n, T_n/I_n, T_nM, T_nMI, etc.

Similarity relations

Z relation (§ 3.6, p. 112)
The all-interval tetrachords:
4-Z15 (0146) & 4-Z29 (0137)
Z-correspondents

Complement relation (§ 3.7, p. 115)
Aggregate - *Any collection containing all 12 pitch classes*
Literal complement
Abstract complement
Complement Theorem
- Proportional distribution of ic (§ 3.7.1)
- Same degree of symmetry (§ 3.7.3)

Hexachord Theorem
- Self-complementary, or Z-related to its complement

IC Vector properties
Unique multiplicity of ic (Ex. 1-22, p. 16)
Maximum ic (e.g., Ex. 3-26, p. 118 – Max. ic 4)
Minimum ic

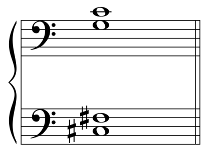
Inclusion relation (§ 3.8, p. 121)
Subsets & supersets
Power set, has 2^c members
Literal subset/superset relations
Abstract subset/superset relations
Inclusion lattice (§ 3.8.2, p. 122)
- Subset classes

Transpositional combination (§ 3.9, p. 124)
TC property

Contour Relations (§ 3.10, p. 126)
CSEG and CSEG-class

EXAMPLE. Stockhausen, *Klavierstück IX*

PC set:



X = (1, 6, 7, 0)

Normal form: [0, 1, 6, 7]

Set class[1]

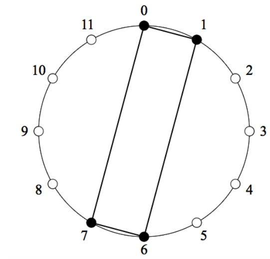
Set class: 4-9 (0167)

IC vector: 200022

DOS: 2, 2

CIA: 1–5–1–5

Polygon notation



Complement:

Literal: *X′* = [2, 3, 4, 5, 8, 9, 10, 11]
Abstract: 8-9 (01236789) 644464

Common tones under T_n and the ic vector:[2]

@	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}
# of common tones	4	2	0	0	0	2	4	2	0	0	0	2

Common tones under I_n and the index matrix (or addition table) and scoreboard:[3]

	0	1	6	7
0	0	1	6	7
1	1	2	7	8
6	6	7	0	1
7	7	8	1	2

@	I_0	I_1	I_2	I_3	I_4	I_5	I_6	I_7	I_8	I_9	I_{10}	I_{11}
# of common tones	2	4	2	0	0	0	2	4	2	0	0	0

Index vector:[4] 242000242000

[0, 1, 6, 7] maps onto itself at T_0, T_6, I_1 & I_7, thus its degree of symmetry (DOS) is: (2, 2)

Literal and Abstract Subsets (2^c):
16 literal subsets: (0, 1, 6, 7), (0, 1, 6), (0, 1, 7), (0, 6, 7), (1, 6, 7), (0, 1), (0, 6), (0, 7), (1, 6), (1, 7), (6, 7), (0), (1), (6), (7), ()
Subset classes: 4-note subset: (0167); 3-note subsets: (016); 2-note subsets: (01), (05) & (06)

References

Straus, Joseph N. 2016. *Introduction to Post-Tonal Theory*, 4th ed. New York: Norton.
Morris, Robert. 1987. *Composition with Pitch Classes*. New Haven: Yale University Press.

Software

Bain, Reginald. *PC Polygon Assistant*. Available online at: <https://reginaldbain.com/software.html>
Buchler, Michael. *Setmaker*. Available online at: <https://myweb.fsu.edu/mbuchler/setmaker.html>.

[1] DOS is *degree of symmetry* (see p. 119). CIA is *cyclic interval array* of the prime form (see Morris 1987, $CINT_1$).
[2] If an entry equals the *cardinality* (*c*) of the set, the set *maps onto itself* at (@) that level of T_n. Every set maps onto itself at T_0.
[3] The *index matrix* shows the inversional *sums* in a *c*-by-*c* matrix. The scoreboard shows the total number (#) of occurrences of each sum in the matrix, which corresponds to the number of common tones @ each level of I_n.
[4] We define the *index vector* to be a listing (without commas) of the number of common tones produced under I_0, I_1, I_2, …, I_{11}.